- **Partial Derivatives**:
  - Take the derivative of one variable while holding all else constant.
  - $\frac{\partial}{\partial x_1} f(x_1, x_2, ... x_n) = \lim_{h \to 0} \frac{f(x_1 + h, x_2, ... x_n) - f(x_1, x_2, ... x_n)}{h}$
  - Notation: $\frac{\partial}{\partial x_1} f(x_1, x_2, ... x_n) = \frac{\partial f}{\partial x_1} = f_{x_1}$
  - Second order partials
- **Differentiability**: A function $f(x_1, x_2, ... x_n)$ is differentiable at $(a_1, a_2, ... a_n)$ iff $f_{x_1}(a_1, a_2, ... a_n)$, $f_{x_2}(a_1, a_2, ... a_n)$, ... $f_{x_n}(a_1, a_2, ... a_n)$ all exist and

  $$\lim_{(x_1, x_2, ... x_n) \to (a_1, a_2, ... a_n)} \frac{f(x_1, x_2, ... x_n) - f(a_1, a_2, ... a_n) - \nabla f(a_1, a_2, ... a_n) \cdot \langle x_1 - a_1, x_2 - a_2, ..., x_n - a_n \rangle}{|\langle x_1 - a_1, x_2 - a_2, ..., x_n - a_n \rangle|} = 0$$

- **Chain Rule**
  - Example: Let $z = f(x, y)$, where $x = g(t)$ and $y = h(t)$. Then $\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$.
  - Use tree diagram to help conceptualize.
  - Partial derivative of $f(x_1, x_2, ... x_n)$ with respect to $t := \nabla f \cdot \vec{r}\,'(t)$
- **Implicit Differentiation**
  - Shortcut of the Calc I way.
  - Use chain rule!
  - Let $f(x, y) = k$. $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{dy}{dx} = 0$. $\frac{dy}{dx} = -\frac{\partial f / \partial x}{\partial f / \partial y}$.
  - Extend to *n* variables.
- **Del operator**: $\nabla = \left\langle \frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, ..., \frac{\partial}{\partial x_n} \right\rangle$.
  - Note: ∇ is an **operator**, not an actual function or value.
- **Gradient Vector**: If $f(x_1, x_2, ... x_n)$, then $\nabla f = \langle f_{x_1}, f_{x_2}, ... f_{x_n} \rangle$.
  - Note: ∇f is normal to $f(x_1, x_2, ... x_n) = k$ in *n*-space
    - Applicably, ∇f is normal to level curves and surfaces (use ∇f to find tangent planes and lines).
    - Extend this concept to a function of *n* variables - ∇f is always normal to the level hypersurface $f(x_1, x_2, ... x_n) = k$.
- **Directional Derivatives**: The derivative of $f(x_1, x_2, ... x_n)$ in the direction $\hat{u} = \langle a_1, a_2, ... a_n \rangle$ in *n*-space, where $\hat{u}$ is a unit vector, is $D_{\hat{u}} f = \nabla f \cdot \hat{u}$.